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The New Technical for Solving Third Order Ordinary Differential Equations by Adomian Decomposition method

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Abstract

In order to solve a third order singvlar initial value differential equations, the effectiveness of using a proposed modification to the (ADM) is examined. A new modification of (ADM)is Presented in this study to solve third order ordinary differntial equations(DEs), the effectivential and the efficiency of this method is cemented by many examples. And shows through the table and graphs that the error rate is very slim.

Keywords- “ Adomain Decomposition Method, third-Order, non-linear, differntial equations, initial value ”

1 Introduction

Many researchers were concerned in studying differntial equations (DEs). (DEs) is an equation between specified derivative on an unknown its values and known quantities and functions. Numerous physical events are mathematically represented as boundary value problems coupled to non-linear third order or ordinary differential equations of the type that are found in physics, engin eering, and other branches of science.Many physical laws are most simply and naturally formulated as (DEs). Singular problem was studied by diferent authers [9, 14, 17] by using (ADM)

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$$y''' + (n + \frac{a}{x})y'' = f(x, y), \tag{1}$$

third-order (DEs) arise in a variety of different areas of applied mathematics and physics, e. g , in the deflection of a curved beam having a constant or varying cross section, a three-layer beam, electromagnetic waves or gravity driven flows and so on [12].

Recently, third-order singvlar initial value problems (BVPs) have received much attention. For example, [10, 11, 15] discussed some third-order two-point BVPs, while [7, 8] studied some third-order three-point BVPs.
third ordinary differential equations and anther was studied by diferent authers [1, 2, 5, 13, 16] .

2 The Adomian Decomposition Method

The ADM involves separating the equation under investigation into linear and nonlinear portion. The linear operator representing the linear portion of the equation is inverted and the linear operator is then applied to the equation. Any given conditions are taken into consideration. The nonlinear part is decomposed into a series of what is called Adomian Polynomials. The method generates a solution in the form of a series whose terms are determined by a recursive relationship using the Adomian Polynomials. A brief outline of the method is a follows. Consider a general nonlinear differential equation as.

$$F(y) = g(x, y), \tag{2}$$

where F is the nonlinear differential operator, y and g are functions of x. In operator form equation (2) is

$$L(.) = x^{-a}e^{-nx}\frac{d}{dx}xe^{nx}\frac{d^2}{dx^2}, \tag{3}$$

with n=1,a=2.

$$L^{-1}(.) = \int_0^x \int_0^x x^{-a}e^{-nx} \int_0^x x^a e^{nx}(.)dxdxdx. \tag{4}$$

$$Ly = g(x) - F(x, y). \tag{5}$$

By solving (5) for Ly

$$L^{-1}Ly = L^{-1}g(x) - L^{-1}F(x, y). \tag{6}$$

For initial value problems we conveniently define L^{-1} for $L = \frac{d^n}{dx^n}$ as the$n$-fold definite integration from 0 to x. If L is a third-order operator, L^{-1} is a three fold integral and by solving (6) for y , we get

$$y = \phi(x) + L^{-1}g(x) + L^{-1}F(x, y). \tag{7}$$

$$\phi(x) = y(0) + xy'(0). \tag{8}$$

Where $y(0)$ and $xy'(0)$ are constants of integration and can be found from the initial conditions.
The Adomian method consists of approximating the solution of (2) as an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{9}$$

and decomposing the non-linear operator N as

$$F(x, y) = \sum_{n=0}^{\infty} A_n(x). \tag{10}$$

Where A_n are Adomian polynomials [3, 4, 6] of $y_0, y_1, y_2, \ldots, y_n$ given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F(\sum_{i=0}^{\infty}\lambda^i y_i)\right]_{\lambda=0}, n = 0, 1, 2, 3, ... \tag{11}$$

Which gives the first four terms A_n are :

$$A_0 = F(y_0),$$

$$A_1 = y_1 F'(y_0),$$

$$A_2 = y_2 F'(y_0) + \frac{1}{2!} y_1^2 F''(y_0),$$

$$A_3 = y_3 F'(y_0) + y_1 y_2 F''(y_0) + \frac{1}{3!} y_1^3 F'''(y_0), \tag{12}$$

and so on

Substituting (7) and (9) into (10) yields

$$\sum_{n=0}^{\infty} y_n = \phi(x) + L^{-1} \sum_{n=0}^{\infty} A_n. \tag{13}$$

The recursive relationship is found to be

$$y_0 = \phi(x) + L^{-1} g(x),$$

$$y_{n+1} = -L^{-1} A_n, n = 0, 1, 2, ...$$

which gives

$$y_0 = \phi(x) + L^{-1} g(x),$$

$$y_1 = L^{-1} A_0,$$

$$y_2 = L^{-1}A_1,$$
$$y_3 = L^{-1}A_2, \tag{14}$$

Addition the plan (12) with (13) can enable us determine $y_(x)$ and hence the series solution of $y(x)$ defined by (9)follows directly. For numerical use the $n$term approximate

$$\psi(y) = \sum_{i=0}^{n} y_i, \tag{15}$$

can be used to approximate the exact sxact solution. The approach above can be support by testing it on a variety of seveal and nonlinear BVP.

3 Numerical Examples

At this point,some examples will be discussed , when n=1 and a=2, in a differential operator (3). The following examples to demonstrate of the accuracy the solution.

3.1 Example 1 :

The equation (1) an example,

$$y''' + (1 + \frac{2}{x})y'' = ((2 + \frac{2}{x})e^x - x) + Lny, \tag{16}$$

with $y(0) = 1, y'(0) = 1, y''(0) = 1$, the equation (16) rewritten as

$$Ly = (2 + \frac{2}{x})e^x - x + Ln(y),$$

added $L^{-1}$to both sides and get

$$y = \phi(x) + L^{-1}((2 + \frac{2}{x})e^x - x) + L^{-1}(ln(y)).$$

And $\phi(x) = (1 + x)$,
then

$$y_0(x) = \phi(x) + L^{-1}(e^x(2 + \frac{2}{x}) - x),$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{17\,x^4}{48} + \frac{81\,x^5}{400} + \frac{89\,x^6}{900} + \frac{373\,x^7}{8820} + \frac{8209\,x^8}{564480}, \tag{17}$$

$$y_1 = \frac{x^4}{48} - \frac{x^5}{400} + \frac{23\,x^6}{10800} - \frac{61\,x^7}{211680} - \frac{221\,x^8}{564480}, \tag{18}$$

$$y_2 = \frac{x^7}{14112} - \frac{x^8}{18816}, \tag{19}$$

from, (17), (18)and(19)get

$$y = 1 + x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{3\,x^4}{8} + \frac{x^5}{5} + \frac{1091\,x^6}{10800} + \frac{4453\,x^7}{105840} + \frac{3979\,x^8}{282240}, \tag{20}$$

Table :The comparison between exact solution $y(x) = e^x$. And ADM.

x	Exact	ADM	Absolut error
-0.9	0.40657	0.30807	0.0985
-0.8	0.22554	0.372087	0.08253
-0.7	0.496585	0.439156	0.057429
-0.6	0.548812	0.50882	0.039992
-0.5	0.606531	0.580992	0.025539
-0.4	0.67032	0.655906	0.014414
-0.3	0.740818	0.734117	0.006761
-0.2	0.818731	0.816542	0.002189
-0.1	0.904837	0.904536	0.000301
0.0	1	1	0.000000000
0.1	1.10517	1.10554	0.00037
0.2	1.2214	1.22467	0.00327
0.3	1.34986	1.36211	0.01825
0.4	1.49182	1.52414	0.03232
0.5	1.64872	1.71915	0.07043
0.6	1.82212	1.95828	0.13616
0.7	2.01375	2.25631	0.24256
0.8	2.22554	2.63281	0.40727
0.9	2.4596	3.11351	0.65391

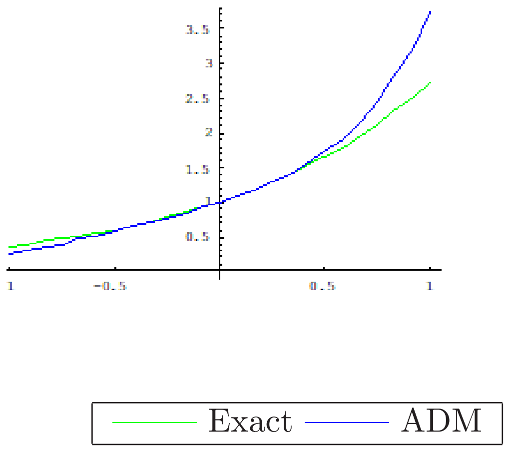

Figure 1: The exact solution $y = e^x$. And the ADM solution $y = \sum_{n=0}^{2} y_n(x)$.

3.2 Example 2 :

A third-order nonlinear equation provide .

$$y''' + (1 + \frac{2}{x})y'' = (2 + \frac{4}{x}) - sin(x) + sin(\sqrt{y}), \tag{21}$$

$y(0) = 0, y'(0) = 0, y''(0) = 2$, the equation (25) is re-written as

$$Ly = e^{-x}x^{-2}\frac{d}{dx}e^{x}x^{2}\frac{d^2}{dx^2}(y),$$

$$= (2 + \frac{4}{x}) - sin(x) + sin\sqrt{y},$$

added $L^{-1}$to both sides and get

$$y = \phi(x) + L^{-1}((2 + \frac{4}{x}) - sin(x)) + L^{-1}(sin(\sqrt{y})).$$

And $\phi(x) = y(0) + xy'(0)$, and the non linear part is

$$y_{n+1} = L^{-1}(A_n), n \geq 0$$

now

$$A_0 = sin(\sqrt{y_0}),$$

$$A_1 = (y_1)(\frac{1}{2\sqrt{y_0}})cos(\sqrt{y_0}).$$

Then

$$y_0(x) = \phi(x) + L^{-1}((2 + \frac{4}{x}) - sin(x)),$$

$$y_0(x) = x^2 - \frac{x^4}{48} + \frac{x^5}{400} + \frac{7\,x^6}{10800} - \frac{x^7}{15120} - \frac{x^8}{80640} + \frac{x^9}{933120} + \frac{11\,x^{10}}{81648000} - \frac{x^{11}}{99792000} + \frac{13\,x^{12}}{574801920}, \tag{22}$$

$$y_1(x) = \frac{1}{212446789632000000}(x^4\ (4425974784000000 + x\ (-531116974080000+$$

$$x\ (-149991367680000 + x\ (16208501760000 + x\ (5030020215000 + 7\,x\ (-92310295000+$$

$$x\ (-20461732588 + x\ (3406073412 + 1306425895\,x)))))))), (23)$$

$$y_2(x) = \frac{1}{2549361475584000000}(-\left(x^6\ (-147532492800000 + x\ (25893457920000+\right.$$

$$x\ (28602062280000 + x\ (-4978283640000 + x\ (-1507609228884+$$

$$x\ (290635178316 + 196653643735\,x)))))))), (24)$$

from (22),(23)and(24)get

$$y = \frac{1}{2549361475584000000}(x^2\ \big(2549361475584000000 + 144074700000\,x^6 - 43697940000\,x^7+$$

$$132285579492\,x^8 - 30071763708\,x^9 - 29256268555\,x^{10})). \qquad (25)$$

Table :The comparison between exact solution $y(x) = x^2$. And ADM.

x	Exact	ADM	Absolut error
0.0	0	0	0.000000000
0.1	0.01	0.01	0.00
0.5	0.25	0.25	0.00
1.1	1.21	1.21	0.00
1.5	2.25	2.25	0.00
2.1	4.41	4.40997	0.00003
2.5	6.25	6.24955	0.000045
3.1	9.61	9.60225	0.000775
3.5	12.25	12.2141	0.0359
4	16	15.81161148	0.18838852

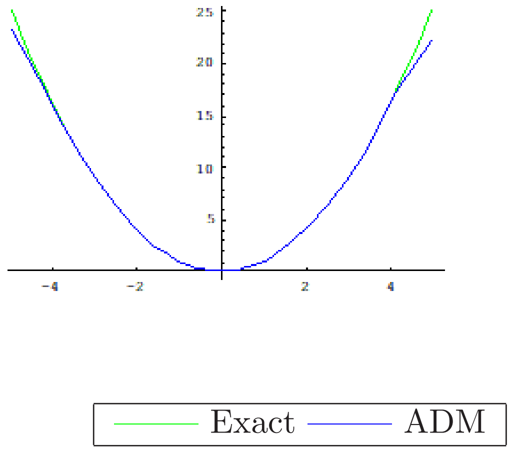

Figure 2: The exact solution $y = x^2$. And the ADM solution $y = \sum_{n=0}^{2} y_n(x)$.

4 Conclusion:

The third-order ordinary differential equations'linear and nonlinear starting value issues were solved using the decomposition approach. In comparison to other methods, the numerical result demonstrated how accurate and efficient this method. In comparison to more time-consuming traditionalprocedures, the decmposition method offers atrustworthy approach that doesn't rely on illogical assumptions, linearization, discretization.

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